

Lecture 7: Frequency & Phase Modulation (FM & PM)

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EE423: Communication Electronics

Angle Modulation (FM & PM)

$$c(t) = A \cos(\omega_c t + \theta_0)$$

$$\varphi_{FM \text{ or } PM}(t) = A \cos \theta(t)$$

$$\omega_i(t) \triangleq \frac{d\theta(t)}{dt}$$

$$\theta_i(t) \triangleq \theta(t) - \omega_c t$$

$\theta(t)$: **generalized angle** of modulated signal.

$\omega_i(t)$: **instantaneous frequency** of modulated signal.

$\theta_i(t)$: **instantaneous phase** of modulated signal.

Frequency Modulation (FM)

- The *instantaneous frequency* of the modulated signal changes in proportion to the message.

$$\omega_{i_{FM}}(t) = \omega_c + k_f m(t)$$

$$\theta_{FM}(t) = \omega_c t + k_f \int_{-\infty}^t m(t) dt$$

$$\varphi_{FM}(t) = A \cos\left(\omega_c t + k_f \int_{-\infty}^t m(t) dt\right)$$

$$\theta_{i_{FM}}(t) = k_f \int_{-\infty}^t m(t) dt$$



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Phase Modulation (PM)

- The *instantaneous phase* of the modulated signal changes in proportion to the message.

$$\theta_{i_{PM}}(t) = k_p m(t)$$

$$\theta_{PM}(t) = \omega_c t + k_p m(t)$$

$$\varphi_{PM}(t) = A \cos\left(\omega_c t + k_p m(t)\right)$$

$$\omega_{i_{PM}}(t) = \omega_c + k_p \frac{dm(t)}{dt} = \omega_c + k_p m'(t)$$



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FM and PM Equivalence

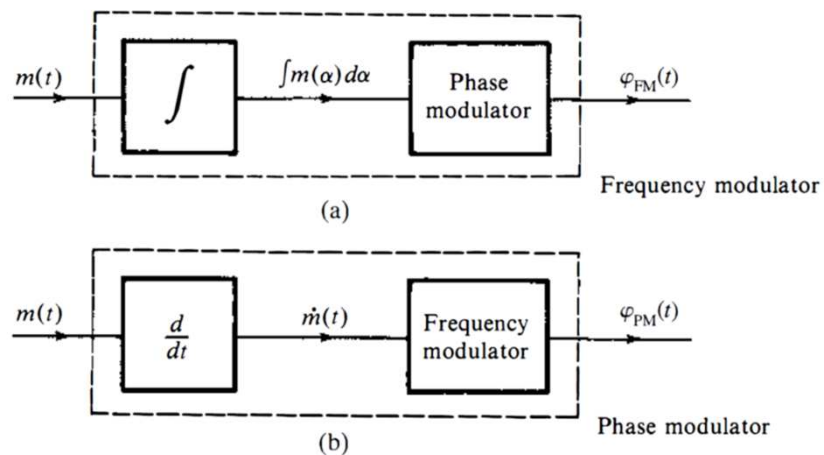
- FM
 - Constant amplitude A
 - Constant carrier frequency ω_c
 - Variable *instantaneous* frequency $\omega_i \propto m(t)$
 - Variable *instantaneous* phase $\theta_i \propto \int m(t)dt$
- PM
 - Constant amplitude A
 - Constant carrier frequency ω_c
 - Variable *instantaneous* frequency $\omega_i \propto m'(t)$
 - Variable *instantaneous* phase $\theta_i \propto m(t)$

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FM and PM Equivalence



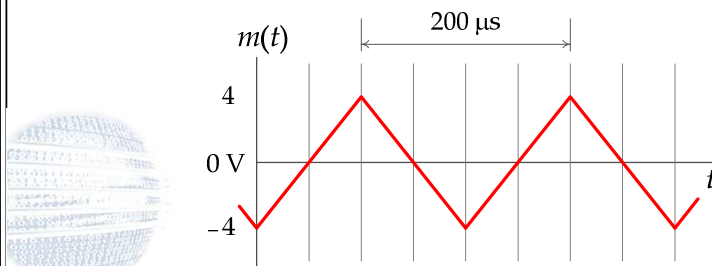
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Example 1

- For the following message signal $m(t)$ and a 100 MHz carrier:
 - Sketch the **FM** modulated signal. Use $k_f = 2\pi \times 10^5$ rad/s/V.
 - Sketch the **PM** modulated signal. Use $k_p = 5\pi$ rad/V.
 - Find Δf for both modulated signals.

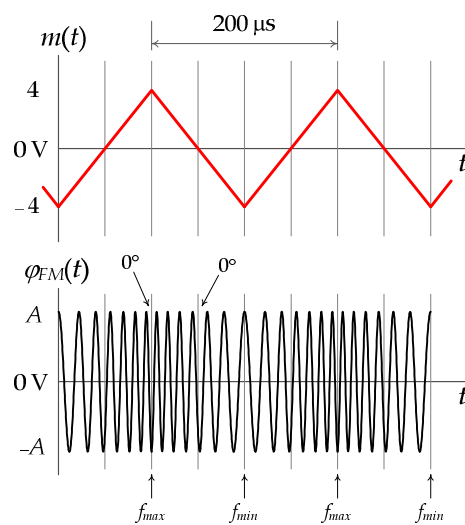


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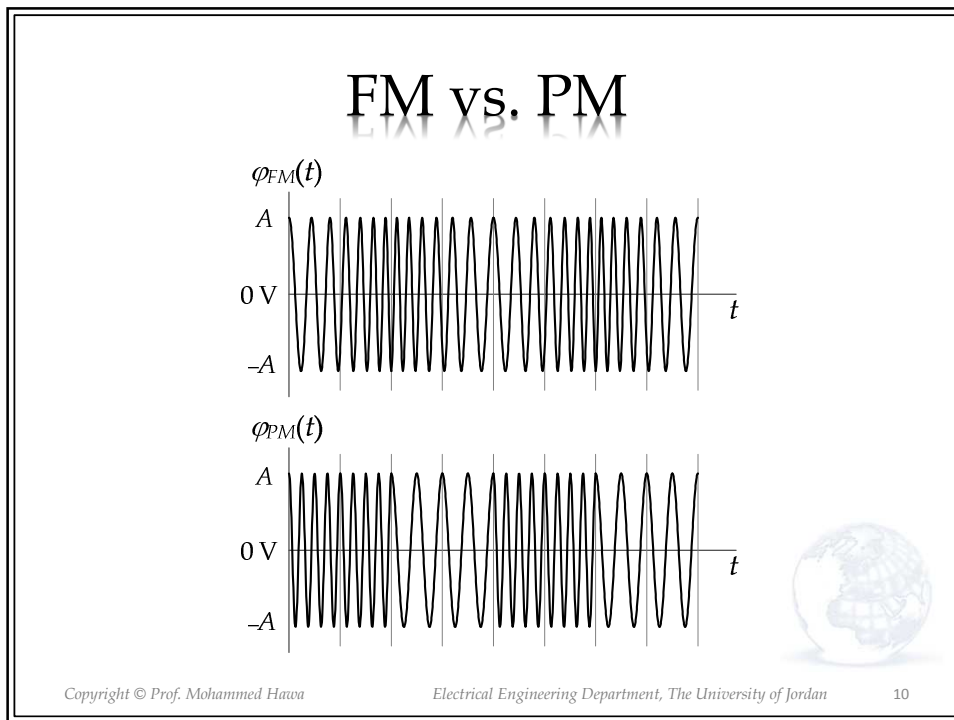
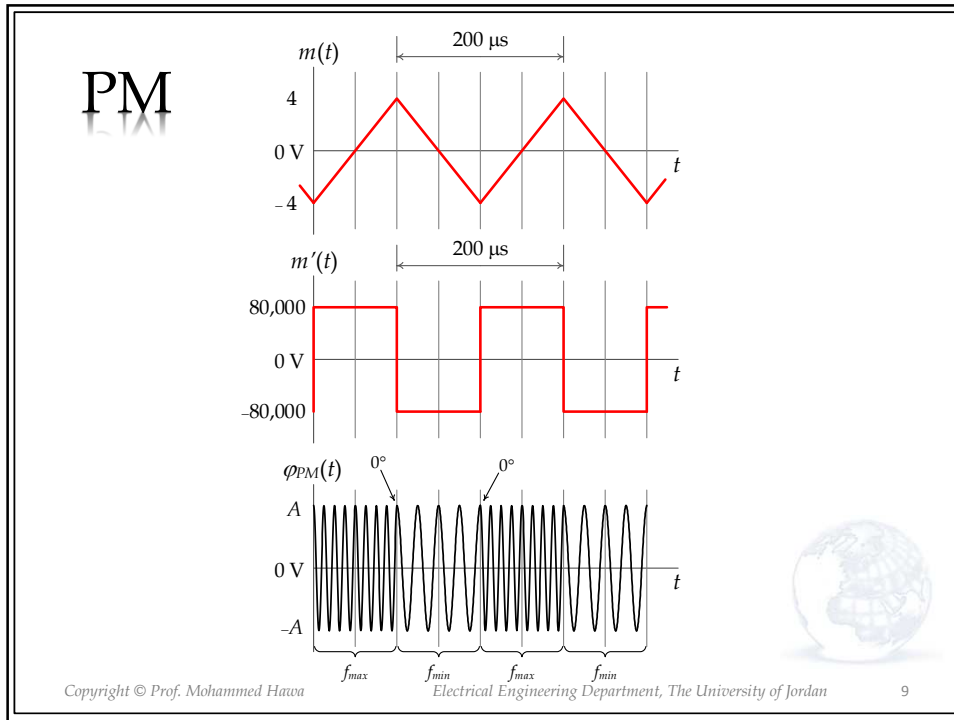
Solution: FM



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Peak Frequency Deviation

- For **FM**:

$$\Delta f \triangleq \frac{f_{max} - f_{min}}{2} = \frac{k_f}{2\pi} \times \frac{m(t)_{max} - m(t)_{min}}{2}$$

$$\Delta f = \frac{k_f}{4\pi} \times m(t)_{pk-pk} \quad [Hz]$$

- For **PM**:

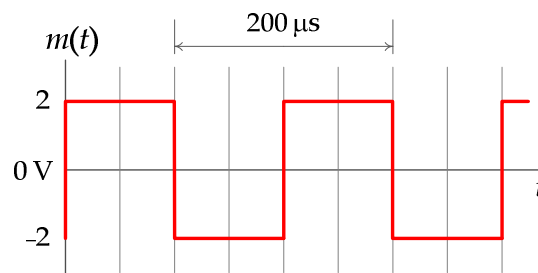
$$\Delta f \triangleq \frac{f_{max} - f_{min}}{2} = \frac{k_p}{2\pi} \times \frac{m'(t)_{max} - m'(t)_{min}}{2}$$

$$\Delta f = \frac{k_p}{4\pi} \times m'(t)_{pk-pk} \quad [Hz]$$

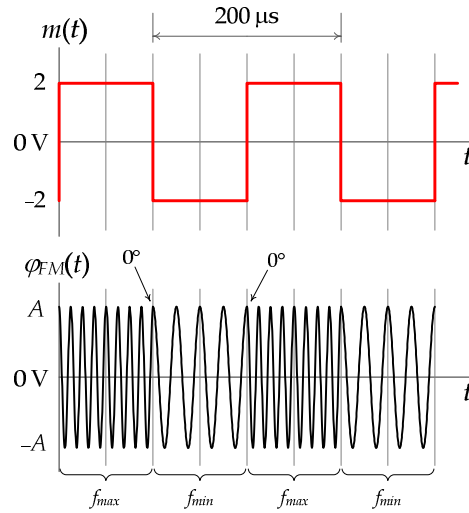


Example 2

- For the following message signal $m(t)$ and a 100 MHz carrier:
 - a) Sketch the **FM** modulated signal. Use $k_f = 2\pi \times 10^4$ rad/s/V.
 - b) Sketch the **PM** modulated signal. Use $k_p = \pi/4$ rad/V.
 - c) Find Δf for both modulated signals.



Solution: FM or FSK

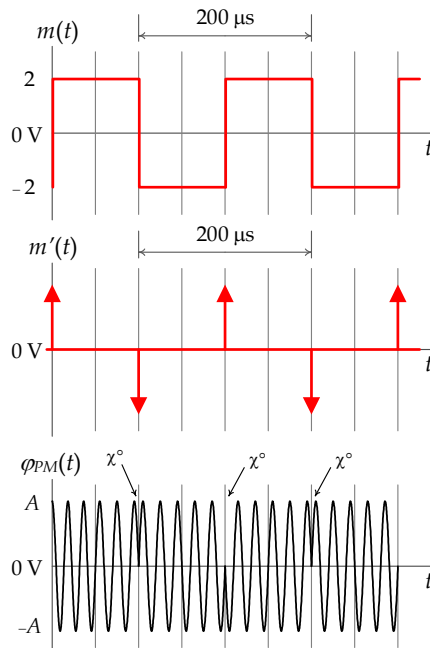


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PM or BPSK



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Rules of Thumb

- Smooth change in instantaneous frequency *always* means smooth change in instantaneous phase.
- Sudden change in instantaneous frequency (i.e., unit step change) *does not* mean a sudden change in phase, i.e., it means 0° sudden phase shift.
- *Impulse* change in instantaneous frequency (i.e., infinity frequency) *might* cause a sudden change in phase. To determine the sudden phase shift (or lack thereof) see $k_p m(t)$ for PM or $k_f \int m(t) dt$ for FM.

FM Hardware

- FM Modulator:
 - VCO (Voltage-Controlled Oscillator).
- FM Demodulator:
 - FM Discriminator (also called Slope Detector or Ratio Detector).
 - Quadrature Detector.
 - Phase-Locked Loop.